

HERIVELTO MARTINS BORGES FILHO (ICMC-USP, BRAZIL)

Exploring Rational Points on Curves over Finite Fields

In this mini-course, we delve into key aspects of curves over finite fields, drawing from a curated selection of topics. In particular, we discuss bounds for the number of rational points, their improvements, and applications. Throughout the five lectures, several open problems will be discussed. Overall, participants will gain insights into the following subjects:

LECTURE 1

We will provide an overview of the historical aspects that have motivated the study of curves over finite fields. We will present and briefly discuss some of the main results and examples that will be explored in the next lectures.

LECTURE 2

We will discuss the problem of constructing and classifying maximal curves, that is, the curves that attain the Hasse-Weil upper bound. The starting points will be a theorem by Kleiman, Serre, and Tate, applied to the Hermitian curve and its large automorphism group.

LECTURE 3

In this lecture, we address the problem of improving the Hasse-Weil bound, with a focus on the alternative bounds provided by Serre, Ihara, and Stöhr-Voloch.

LECTURE 4

In this lecture, we delve deeper into the theory of Stöhr-Voloch, proving a prototype version of their main results and discussing the notion of curves with many points in this context. Particular emphasis is given to Frobenius classical curves.

LECTURE 5

We discuss the rudiments of the theory of error-correcting codes and the role of curves with many points in the construction of codes with good parameters.

REFERENCES

- [1] Arakelian N., Borges H., "Bounds for the Number of Points on Curves over Finite Fields," *Israel Journal of Mathematics*, 228 (2018), 177-199.
- [2] Hirschfeld, J.W.P., Korchmáros G., Torres F., "Algebraic Curves over a Finite Field," *Princeton Series in Applied Mathematics*, 2008.
- [3] Stöhr K.O., Voloch J.F., "Weierstrass Points and Curves over Finite Fields," *Proceedings of the London Mathematical Society*, 52 (1986), 1-19.