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Higher homological algebra in representation theory

A classical result by Auslander states that there is a correspondence between Artin algebras of finite representation type and algebras whose global dimension at most 2 and dominant dimension at least 2. At the end of the 2000's Iyama realized and proved that this result constitutes a special case of a more general theorem stating a correspondence of "$\mathtt{d}$-representation finite" algebras and algebras whose global dimension at most $\mathtt{d}+1$ and dominant dimension at least $\mathtt{d}+1$. The main insight of Iyama was to realize that, instead of looking at the whole of the module category, a generalization of Auslander's result required considering a maximal orthogonal rigid subcategory, also known as a "$\mathtt{d}$-cluster tilting" subcategory, which behaves much like an abelian category where the shortest non-split exact sequences have $\mathtt{d}+2$ terms. In other words, in a $\mathtt{d}$-cluster tilting subcategory the bifunctor $\mathrm{Ext}^{\mathtt{i}}(-,-)$ is zero for every $\mathtt{i}$ between 1 and $\mathtt{d}-1$.

In this course we will go over the basics on representation theory of Artin algebras, we will give the definition of $\mathtt{d}$-cluster tilting subcategories and we will see how the classical notions adapt to the higher setting. The topics covered in this course include (higher) homological algebra, (higher) Auslander-Reiten theory and (higher) torsion classes.

LECTURE 1

Module categories and homological algebra. In this first lecture we will give a short introduction to homological algebra and we will explain the main properties of module categories of Artin algebras, which is the prime example where higher homological algebra arises.

LECURE 2

Higher homological algebra. In this second lecture (or maybe at the end of the previous lecture) we will discuss the first notions of higher homological algebra ($\mathtt{d}$-cluster tilting subcategories, $\mathtt{d}$-exact sequences, etc.) and we will see in which way higher homological algebra generalizes classical homological algebra.

LECTURE 3

Torsion classes in abelian categories. In this lecture we will speak about an important notion in homological algebra: torsion theories. We will give the definition of torsion theories and we will discuss some of its key properties. Later we will concentrate on torsion theories in module categories and its relationship with some distinguished objects known as τ-rigid objects.

LECTURE 4

Torsion classes in $\mathtt{d}$-abelian categories. Mirroring what we have done in the first part of the course, in this fourth lecture we will consider $\mathtt{d}$-torsion classes in $\mathtt{d}$-cluster tilting subcategories in arbitrary abelian categories and also in module categories, where we will see its relationship with $\tau_{\mathtt{d}}$-rigid objects.

LECTURE 5

Combinatorial aspects of higher homological algebra. In this last talk we will introduce a family of algebras known as higher Auslander algebras of type $\mathtt{A}$ and we will explain how to understand its $\mathtt{d}$-cluster tilting subcategory combinatorially. Then we will use this description to count explicitly different notions of interest in higher homological algebra.