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## **Vertex Algebras**

This minicourse is an introduction to vertex algebras. Other than a general background in algebra, no specific prerequisites are required. We adopt the following point of view: the study of infinite dimensional Lie algebras and their representations requires methods that have no analogues in the finite dimensional world. These techniques (which are secretly related to ideas from quantum field theory) turn out to lead naturally to the notion of vertex algebra.

### LECTURE 1

In this lecture we briefly review finite dimensional simple Lie algebras and their highest weight representations. Then we familiarise ourselves with some important infinite-dimensional analogues: Kac-Moody, Heisenberg, and Virasoro Lie algebras.

### LECTURE 2

In this lecture we investigate a way to construct representations of the Virasoro Lie algebra from representations of the Heisenberg Lie algebra (the latter being much easier to understand). Then we see how this construction leads to the notion of vertex algebra.

### LECTURE 3

In this lecture we study the notion of vertex algebra, get to know examples related to the infinite dimensional Lie algebras of the first lecture as well as examples coming from integral lattices, and learn how to do calculations in these algebras.

### LECTURE 4

In this lecture we use lattice vertex algebras to give a proof of an important result in representation theory of infinite dimensional Lie algebras: the Kac determinant formula.

### LECTURE 5

In this lecture we return to the historical origins of vertex algebras, and learn what they have to do with monstrous moonshine.