

## Torsion free modules over commutative domains of Krull dimension 1

Let  $R$  be a commutative domain. Let  $\mathcal{F}$  be the class of  $R$ -modules that are infinite direct sums of finitely generated torsion-free modules. In the talk we will discuss the question whether  $\mathcal{F}$  is closed under direct summands.

If  $R$  is local of Krull dimension 1, we prove that  $\mathcal{F}$  is closed under direct summands if and only if any indecomposable, finitely generated torsion-free module has local endomorphism ring. If, in addition,  $R$  is noetherian this is further equivalent to  $R$  having local integral closure.

Recall that  $R$  is domain of finite character, if any nonzero ideal of  $R$  is only contained in a finite number of maximal ideals. For domains of finite character and of Krull dimension 1, the property  $\mathcal{F}$  being closed under direct summands is inherited by localization at a maximal ideal and, moreover, any localization at a maximal ideal of  $R$ , except may be one, satisfies that any finitely generated ideal is two-generated. We manage to prove that the converse is true when  $R$  is integrally closed and also when  $R$  is noetherian with module-finite normalization.

The proof of such results rely on two main technical tools:

- Příhoda's theory of fair-sized projective modules [4,3] and its extension to a non-noetherian setting [2], that gives us a way to construct infinitely generated non-trivial summands;
- The Package Deal Theorems by L. Levy and C. Odenthal [5] that we extend to the setting of  $\mathfrak{h}$ -local domains that allows to glue together in a module over  $R$  compatible families of torsion-free modules over the localizations at maximal ideals.

These results are contained in the preprint [1].

### REFERENCES

- [1] R. Álvarez, D. Herbera, P. Příhoda, *Torsion-free Modules Over Domains of Krull Dimension One*. Preprint, 2024.
- [2] Herbera, D., and Příhoda, P. *Reconstructing projective modules from its trace ideal*. Journal of Algebra, **416** (2014), pp. 25–57.
- [3] Herbera, D., Příhoda, P. and Wiegand, R. *Big pure-projective modules over commutative noetherian rings: comparison with the completion*. Preprint (2023); arXiv:2311.05338.
- [4] P. Příhoda, *Fair-sized projective modules* Rend. Semin. Mat. Univ. Padova **123** (2010), 141–167.
- [5] Levy, L. S., Odenthal, C. J. . *Package Deal Theorems and Splitting Orders in Dimension 1*. Transactions of the American Mathematical Society, **348** (9) (1996), 3457–3503.